"***The basics of Theory of Gas and Heat Airships***" is unique work written by Lev KONSTANTINOV and it will be published in the journal. The work of uncommon beauty and elegance includes theory problems and practical recommendations. The work was written for all those who will simultaneously to ply a theory of airship and its making.

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"The Basics of Gas and Heat Airship Theory "

Introduction

Controlled lighter than air (LTA) aircrafts naming as airships and appeared at the end of the nineteenth century had short but bright history of its development and it was demonstrated brilliant possibilities of practical application.

"Dinosaurs of Air " – gigantic vehicle with length about quarter of a kilometer and carried almost 100 tons cargo and transported it on distance evaluated about dozens thousand kilometers, terminated its existence to the end of 30-s of the twentieth century. Like their co-brothers in animals, who were died and left as a memory about themselves only small reptiles, we can see now not so big number of small, low speed, low altitude airships of most imperfect non-rigid type without exact efficiency function.

Did the airships lose their meaning or they still have any future? The question is discussing widely in mass media and scientific conferences. Enthusiast's argument to adduce proof in favor of airship*,* the airships ecological niche was defined as a field of low flying altitudes and speeds for cargo delivery to regions, which are not available for traditional commercial aviation. But the deal is not in progress. The emotional articles of the enthusiasts do not find financial support of governments. The airship industry development has to resolve the economical problems, but without government support it is impossible.

As ever as always the first step is difficult, especially after unsuccessful experience of dozens airships' catastrophes in 30-s of the twentieth century, which compromises substantially the idea of the flights by airship.

Use of previous experience of airship design and construction cannot be forceful basis for airship industry regeneration, even with application of newest achievements in science, technique and technology. Rigorous and ensemble theory, which can predict realistic possibilities of controlled LTA vehicles, needs to be developed. The theory cannot colour the advantages and must not hide the disadvantages of airships.

F. Engels affirmed the following: "There is nothing more practical than good theory."

And such theory generalizing experience of the past and achievements of present, must be put into the basics of true analysis and practical conclusions, which are required for making and operational using of airships on the new basics. The application of the theory will terminate castle-building about super gigantic airships equipped with atomic engines, about "flying cities" and so on. The theory will allow on the basics of strong scientific approaches to optimization of designs, to define most rationalized design decisions and parameters of airship maintenance for different purposes. The theory will evaluate economic result and area of real airships marketability in comparison with heavier than air aircrafts. The achievements of modern aerostatics, aerodynamics, theory of strength, heat transfer, theory of aviation engines, methods of mathematical modeling and optimization of designs and processes, applied to which the LTA vehicles must be used for those purposes as well.

This work will not decide hardly all problems, but the main direction is marked by it and it gives rather clear understanding about results that can be achieved in design and maintenance of the airships.

Part 1

The Theory of Gas-filled Airships.

§1. Basics of Aerostations.

A use of lifting gas as a lift source is based on Archimedes' principle that defines lift P_L, caused by the difference between densities of the air ρ_A and lifting gas ρ_G, as:

$$P_L = V(c_A - c_G) = V\Delta\rho, \text{ kg} \quad (1.1)$$

where: V – value of the gas in envelope of an airship (aerostat), m^3;
ρ_A, ρ_G – air and gas densities, kg/m^3;
$\Delta\rho$ – specific buoyancy of 1 m^3 of gas;

The densities of the air and gas can be defined rather exact according to Clapeyron equation for Ideal Gas State:

$$pv = RT; v = \frac{RT}{p} = \frac{1}{\rho}; \rho = \frac{p}{RT}, \quad (1.2)$$

where: v – specific gas value, m^3/kg;

R – gas constant, kgm/kg °C (29.27 for air, 424 for hydrogen, 212 for helium);
T – absolute temperature, °K;
P – atmospheric pressure, kg/m^2.
Values of different gas constants can be bounded up with its molecular mass *μ* and with gas universal constant *R* = 848 kgm/kg °C, by ratio:

R_i = *848/ μ, kgm/kg °C;* (1.3)

As a result in LTA vehicles the largest buoyancy can be provided by gas which has the least molecular mass (hydrogen, *μ* = 2.0; helium*, μ* = 4.0, etc.).
Air density ρ_A is variable value and it depends on atmosphere condition, season and altitude. Because it is rather difficult to take into consideration all mentioned parameters, the conditional state of atmosphere is used for calculation. The conventional state of atmosphere is defined by equations of International Standard Atmosphere (ISA). The mentioned equations till altitude of 11000 m have such a view:

$$\frac{p}{p_0} = \left(1 - \frac{H}{44300}\right)^{5.256}; \frac{\rho_H}{\rho_0} = \Delta = \left(1 - \frac{H}{44300}\right)^{4,256} \quad (1.4)$$

t_H = 15-0.0065H, °C; $P_0 = B_0$ = 760 mm. = 10331.7 kg/m^2,
where:
H – altitude, m;
t_H – temperature at altitude H, °C;
B_0 –atmospheric pressure at sea level, kg/m^2.
The main values defined ISA and specific buoyancy of 1 m ^{3} of hydrogen and helium till the altitude 5000 m are shown in the Table 1; values: ν – air cinematic viscosity coefficient, m^2/s*;* λ_A– air heat transfer coefficient, Wt / (m·K°).

ISA table till altitude 5000 m. *Table 1*

H m	p mm	T °K	Δ	$\nu \cdot 10^6$ m^2/s	$\lambda_A \cdot 10^2$ Wt / (m·K).	$\Delta\rho = \rho_A - \rho_G$	
						hydrogen	helium
0	760,00	288,00	1,0000	14,57	2,55	1,141	1,056
500	715,98	284,75	0,9528	15,15	2,52	1,087	1,007
1000	674,06	281,50	0,9074	15,77	2,50	1,035	0,959
1500	634,14	278,25	0,8636	16,42	2,48	0,985	0,912
2000	596,15	275,00	0,8215	17,20	2,45	0,937	0,868
2500	560,04	271,75	0,7810	17,82	2,43	0,891	0,825
3000	525,71	268,50	0,7420	18,60	2,40	0,847	0,784
3500	493,11	265,26	0,7045	19,45	2,38	0,804	0,744
4000	452,16	262,00	0,6685	20,30	2,35	0,763	0,706
4500	432,81	258,75	0,6339	21,20	2,33	0,723	0,670
5000	404,99	255,50	0,6007	22,10	2,30	0,685	0,635

Airships and aerostats are divided on fulfilled and unfulfilled depending on how their gasholders are filled up. First ones, have gas-holders filled up with gas in full value, that equals to maximum calculated value *V*. Unfulfilled airships and aerostats have gas-holders filled up with gas partially and value of gas equals to *V´*, that is less than value *V*. A ratio $\overline{V} = V'/V$ is called as a degree of admission and it characterizes the altitude capability and load-carrier capacity of LTA vehicles.
Fulfilled aerostat having constant value V has considerable aerostatic lift excess ΔP_c (buoyancy) near the earth level and it does not depend on calculated altitude of aerostat flight. Buoyancy is a difference between lift and gross weight of the vehicle including payload G_Σ:

$$\Delta P_{c0} = V\Delta\rho_0\Delta - G_\Sigma,\ kg \quad (1.5)$$

The buoyancy at an altitude H defines as:

$$\Delta P_{cH} = \nu\Delta\rho_0 - G_\Sigma,\ kg \quad (1.6)$$

where: $\Delta\rho_0 = \rho_A - \rho_G$ at H = 0; $\Delta = \rho_P/\rho_0$ by ISA.
The limiting altitude of fulfilled aerostat flight defines of condition

$$\Delta P_{cH} = 0 \text{ or } V\Delta\rho_0\Delta = G_\Sigma, \quad (1.7)$$

Due to gas expansion during aerostat climbing this process is accompanied with blowing of lifting gas and discharging of ballast that balanced the aerostat at ground level.
Total gas mass lost during climbing from altitude H=0 till H_i equals:

$$\Delta G_G = v(1-\Delta)\rho_0, kg \quad . \tag{1.8}$$

Weight of discharged ballast must be equal to aerostatic lift of gas blown of the envelope [1]. Obviously, such a gas loss is not reasonable. That is why the application of unfulfilled airships is more suitable due to fulfilling comes to its end at the calculated flight altitude *H.*
The buoyancy of lifting gas at the altitude H and under known maximum value V equals to:

$$P_{L\,H}=V\Delta\rho_0\Delta=G_x,\ kg. \tag{1.9}$$

In case of unfulfilled gas-holder at ground level (H=0) the mass of gas is:
$G_G=V'\rho_H=\Delta\rho_0\Delta$, kg and its value is V':

$$P_{L0}=V'\Delta\rho_0=V\Delta\rho_0\Delta=P_{LH}=G_\Sigma,\ kg. \tag{1.10}$$

Thus, aerostatic lift of unfulfilled airship equals to weight of the airship and remains constant during climbing from the ground level H=0 till maximum calculated altitude $H_{max.}$. In this case the degree of inflation is V=Δ and it means that airship is in indifferent equilibrium state, and additional buoyancy ΔP_c needs to be applied for climbing up to the altitude H.
Unfulfilled airship indifferent equilibrium under low altitude flight near the surface creates big difficulties for airship flying and may result in catastrophe. Changing of air temperature and pressure along the flight path, ascending or descending currents creating negative buoyant can make for the collision of the dirigible with earth or water surface. A few of airship catastrophes were caused apparently by mentioned above reason (catastrophes of airship “Italy” in 1928, American airships “Akron” and “Macon” in 1933 and 1935, Soviet airship B-6 in 1938 and so on).
Additional buoyancy ΔP_c can be created by different methods as following:
1) - adding of lifting gas into gas holder;
2) - ballast dropping;
3) - aerodynamic method by means of propellers creating vertical directed thrust or by elevators after accelerating to required speed near the earth.
The method number one: the gas holder fills fully when the airship arrives the limited altitude. As a result redundant gas creating excess buoyancy needs to be discharged to avoid surplus pressure into the envelope.
The method numbers two: ballast drop permits the climbing till given altitude without loss of lifting gas. But payload will decrease on the value of dropped ballast. The limiting altitude can be increased under such condition (1.7) that can be calculated according to the formula:

$$H = 44300\left(1-\frac{G_\Sigma}{V\Delta\rho_0}\right)^{0.235}, kg\ . \tag{1.11}$$

Negative buoyancy needs to be used for descent of the airship, namely the force must be directed to the earth side. The force can be created by the same methods that are used for airship climbing. Methods without ballast dropping and lifting gas discharging are preferable and they can be realized in such the ways:
1) – lifting gas pumping by compressor into special receiver;
2) – by thrust vector controlled propellers or elevators;
3) – accumulating of condensed from engines exhaust water as ballast on the last step of the flight and dropping it when necessary.

In any of above mentioned cases the dirigible starts to go down. Theoretically in case of aerodynamic resistance absence, the descending of the airship takes place under free fall acceleration. But in reality airship descends with retardation due to air density increasing.

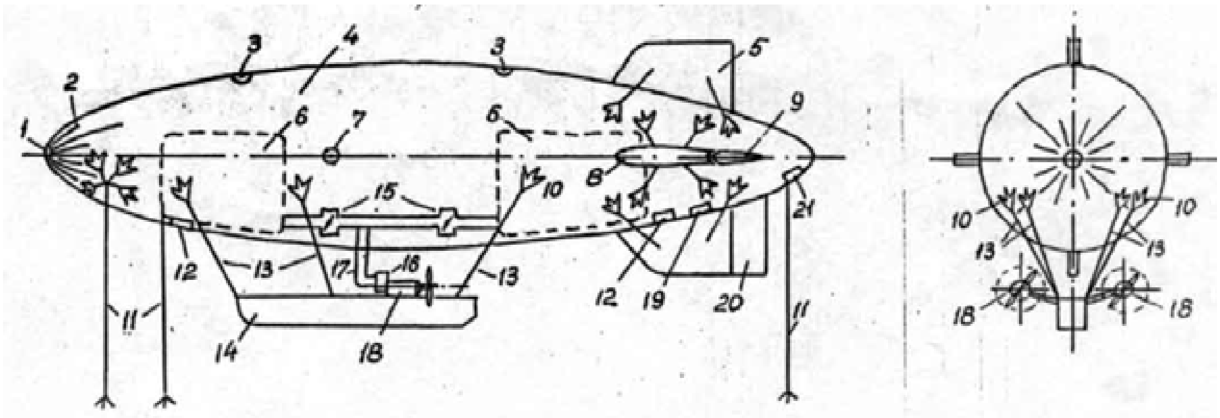
§2 Purpose of Airships and its main types.

Concept of efficiency function of airship can be formulated depending on purpose of the airship for fulfillment these or those goals. The concept of efficiency function can be presented as mathematical expression of airship effect correlated with design, mode and economic factors. The definition of extremum of the mathematical expression of dirigible effect gives possibility to put a problem of airship design optimization.
According to set goals the following main purposes of the airships can be presented:
1. The airships for transportation of given payload for definite distance (cargo, passengers and tourists ones).
2. The airships for maximum range flights without payload (for scientific purposes).
3. The airships for maximum or set endurance without payload (for patrol flights).
4. The airships for short endurance flights with payload (sightseeing and entertainment flights with passengers, transportation of wood from hard-to-reach areas or heavy and large cargo, and so on).

Besides of above mentioned main purposes, there can be another ones that destined for fulfillment this or that special aim. Realization of airship aim function is bounded up with design peculiarity depending of which airships are subdivided on flexible, semi-rigid and rigid types.

Flexible airship (Fig.1) *Fig.1 Flexible airship scheme.*

1- *mooring point; 2- bow stiffening, 3- rip panel; 4- flexible envelope; 5- fin; 6- ballonets; 7- controlled valves; 8- stabilizer; 9- elevators; 10- mounting foot; 11- mooring lines; 12- safety valves; 13- cords; 14- gondola; 15- valve; 16- fan; 17- air duct; 18- power plant; 19- safety valve; 20- rudder; 21- appendix.*



has flexible envelope – 4 manufactured of rubberized fabric. Gondola -14 with power plant -18 is attached to the envelope by means of sewn or glued mounting foots -10 and cords -13. Fin -5, stabilizer –8, elevators -9 and rudder -20 form the tail unit. Special ballonets -6 are used to keep the inalterable shape of airship under changes of atmosphere pressure and temperature. The ballonet is manufactured of flexible shell and is pumped with air by special fan -16. The safety valves –12 are used to blow down gas from the gas holder under increased pressure. Controlled valves -7 are used to blow down the part of gas from the gasholder and thus to descent the airship. There is also the rip panel -3 to discharge all the gas as quickly as it is possible after landing in emergency.
Flexible airship has became base for development of semi-rigid airship (Fig.2).

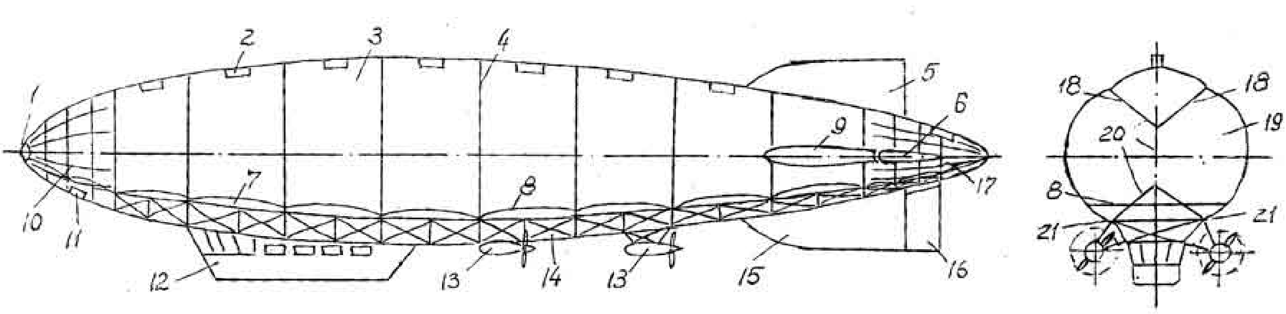


Fig.2. Semi-rigid airship scheme.
1 - mooring point; 2 - safety valves; 3 - envelope; 4 - pressure baffle; 5 - upper fin; 6 - elevators; 7 - ballonets; 8 - diaphragm; 9 - stabilizer; 10 - bow stiffening; 11 - inlet duct; 12 - crew gondola with passenger car; 13 - engine pod; 14 - keel beam; 15 - bottom fin; 16 - rudder; 17 - tail stiffening; 8- belts; 19 - gas holders; 20 - cords; 21 - pivoted point of fin beam.

Italian designer Umberto Nobile in his conception of airship design (1920) realized the most perfect embodiment of semi-rigid airship system. Presence of keel beam –14 inscribed into the contour of the airship is the typical for semi-rigid airship. The keel beam is a truss of triangular cross-section. The keel beam has also the reinforced units at nose –10 (bow stiffening) and at rear 17 (tail stiffening) to accept the increased loads caused by raised flight speed and thus to keep the inalterable shape of nose and tale parts. The envelope volume is divided into separated sections by vertical flexible pressure baffles –4. Then every section is divided in two parts. Upper part is filled up by lifting gas. Like to ballonet, lower part is filled up by incoming air flow through inlet duct –11. Every cell of gasholder and ballonet is equipped with safety valves that can operates as controlled one also. Crew gondola with passenger car –12 and engine pods –13 are attached to the bottom of keel beam. The fins –5, 15, stabilizers –9, rudders and elevators -16, -6 are designed like to flexible airship.

German designer Ferdinand Von Zeppelin in 1930 (Fig.3)

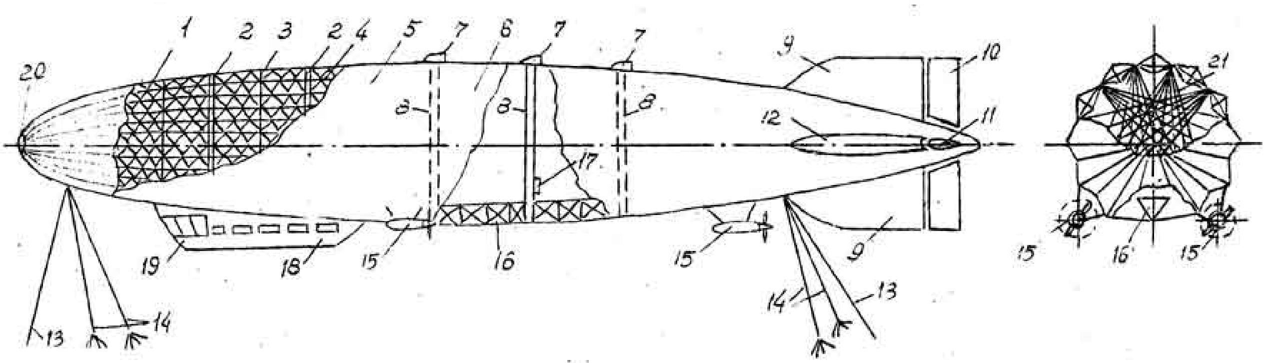
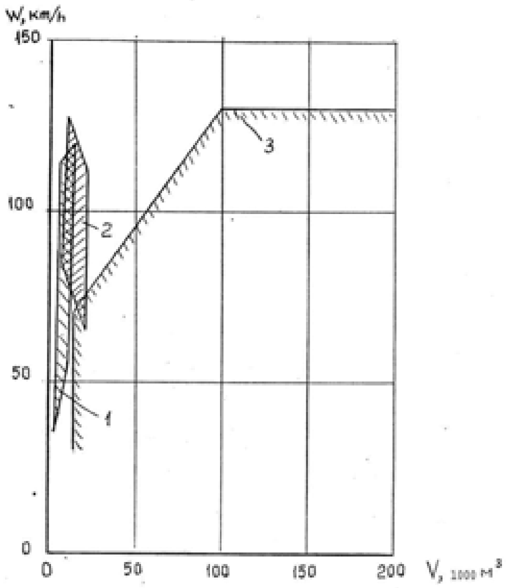


Fig.3. Rigid airship system.
1- rigid framework; 2- main frames; 3- secondary frames; 4- stringers; 5-envelope; 6- gasholders; 7- control valves; 8- vertical gas passage; 9- fin; 10- rudder; 11- elevator; 12- stabilizer; 13- tie-down cables; 14- cables for ground team; 15- engine pods; 16- corridor; 17- automatic safety valves; 18- passengers car; 19- crew gondola; 20- mooring point; 21- frame bracing

realized most prefect rigid airship system in conception of airship design. The main peculiarity of the design is rigid framework –1, constructed according to transverse-longitudinal scheme consisting of transverse elements as frame (such as main frames –2 and secondary frames -3) and longitudinal elements as stringers -4. The rigid framework carries all external loads and has flexible fabric envelope -5. Internal space of airship is divided into sections by main frames with separated gasholders –6 inside of every section. Buoyant lift transfers to the framework by special nets covering the gasholders. The gasholders are equipped the automatic safety valves –17 and with control valves –7 for maneuvering. The discharged by safety valves gas passes from holders through vertical passage –8 towards the envelope top. There is no any ballonet. Crew gondola -19, passenger car -18 and engine pods 15 are attached to the framework. Control system of the airship is like on the previous types of designs and includes the fin –9, stabilizers –12, rudders 10 and elevators -11. Structure of the airship is supplied with mooring point –20 in the nose part of the framework, corridors (inspection galleries) to pass along all the airship body and consisted of filled by water tanks ballasting system.



§3. Spheres of airship application, advantages and disadvantages of different airship systems.

Spheres of different systems airship application are shown on the Fig.4.

Fig.4. Application fields for different airships` schemes: 1 – flexible; 2 – semi-rigid; 3 – rigid.

The value of flexible airship is changed in limits from 1 till 14 thousand m^3 under flying speed less than 115 … 120 km/h. The advantages of flexible airship are simple design, maximum load factor (payload to gross take-off weight ratio), its low weight. There is possibility of quick disassembly and transportation of the airship. There are few advantages during maneuvering under low flying speed. Partly, there is possibility of quick pitching change by different filling of ballonets.

Nevertheless flexible airships have many disadvantages as well, that reduce to zero all mentioned advantages, namely: impossibility of dirigible making with big gross take-off weight because of complexity of keeping large flexible airship's shape. There are no any methods to distribute all the loads along the envelope surface. Thus it limits flight range and altitude. Flexible airships have low reliability and at speed over 100 km/h are very dangerous because there is probability of its envelope deformation. From the other side the sphere of the flexible airship application becomes smaller dramatically, because the airship hasn't possibility to fly against strong wind.

Advantages of rigid airship are inalterable shape under changing loads and external conditions, reliability and long life. The rigid airship system can supply flight safety, increases flying speed, decreases drag, and improves balancing and controlling. Furthermore rigid airship has few disadvantages, such as: complex framework's structure, difficulties of its construction calculation and designing. The rigid airship system leads to big cost and man-hours at ground servicing and during flying. There is such opinion that good result of zeppelins operation was reached due to smooth cooperation between crews and ground teams.

Nevertheless till now there is such point of view, according to which the zeppelin conception is the only possible one for future commercial airships of big range and load-carrying capacity.

Semi-rigid airships system takes up intermediate position between flexible and rigid airship systems, so disadvantages of flexible airships are peculiar to them in large rate. Structure of big semi-rigid airships came across with essential difficulties. Thus, the best semi-rigid airships designed by U. Nobile – "Norge", "Italy" and Soviet B-6 had value nor more than 18500 m^3 and payload under 8.5 ton. Obvious, such performance cannot be taken into consideration as a prospect for airships of future.

§4. Airships Geometric Characteristics

A shape of gas filled airship body must correspond to its aim function and support necesary load-carrying capacity, flying speed and altitude. It requires no any special arguments, that minimum weight of the airship structure under given buoyant lift can be achieved under sphere-shaped body. But such shape has big drag and that is why classic airship has shape of low drag oblong streamlined body of revolution. Rate of elongation that is called as fineness ratio of the airship body characterizes by ratio $\lambda = L/D_{max}$, where L- length, and D_{max} is maximum diameter of the body.

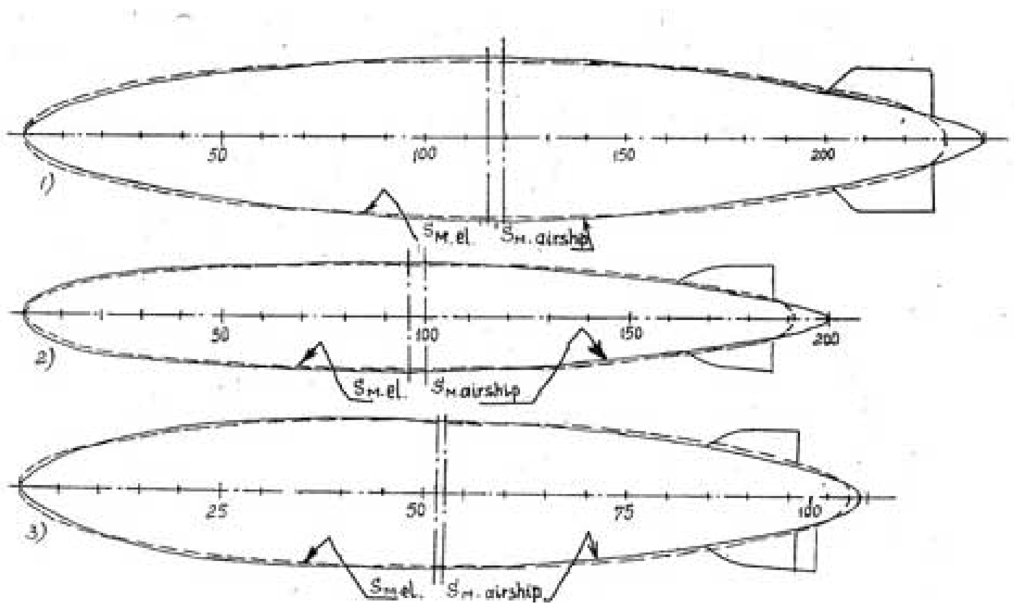


Fig. 5. Comparison of airships (----------) and equivalent ellipsoid of revolution (- - - - - - -) contours.
1) "Akron", V = 184 000 m^3; λ = 5.9; L = 239 m; D_{max} = 40.5 m; L_{ell} = 230.41 m; D_{ell} = 39.053 m;
2) LZ-126, V = 70 000 m^3; λ = 7.246; L = 200 m; D_{max} = 27.6 m; L_{ell} = 191.47 m; D_{ell} = 26.424 m;
3) B-6, V = 18 500 m^3; λ = 5.56; L = 104.5 m; D_{max} = 18.8 m; L_{ell} = 102.985 m; D_{ell} = 18.522 m.

Bodys shape of flaxible, semi-rigid and rigid airships are very close to the characteristics of oblong ellipsoid The main difference between shape of airship body and shape of ellipsoid of revolution is in the tail part, that is characterized by streamlined tail cap. This fact gives possibility to make analysis of different factors, that have an effect on design and aerodynamics of the airship, by use of equations described ellipsoid of revolution. The ellipsoid of revolution is particular case of triaxial ellipsoid equation:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \qquad (1.12),$$

where: *a,b,c,* are semi-axises of ellipsoid.
In case *a=c* and *a<b* the ellipsoid has oblong shape, and if *b=c* and *a<b* the ellipsoid has oblate shape (like to "flying saucer" or disc shaped), (Fig.6).

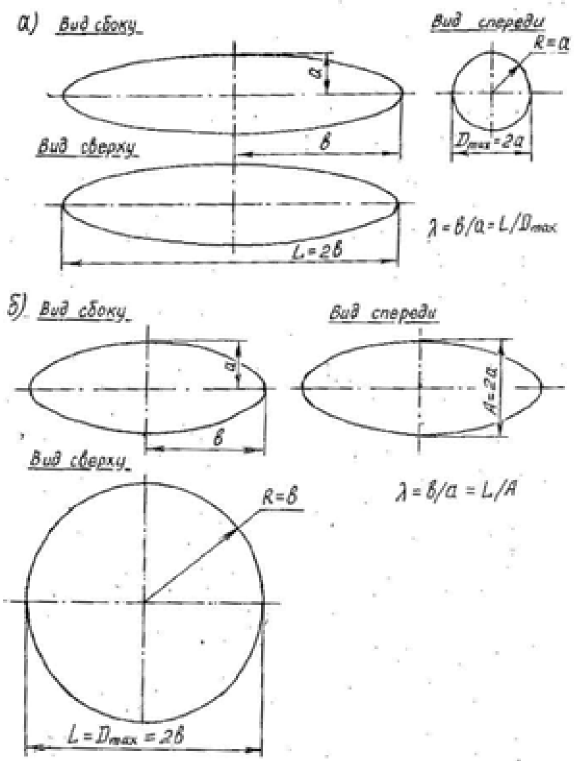


Fig.6. Correlations of geometrical parameters of oblong (a) and oblate (b) shapes of ellipsoids of revolution.

The use of notion about equivalent ellipsoid of revolution is reasonable. In this case the equivalent ellipsoid of revolution has the same value *V* and fineness ratio $\lambda = L/D_{max} = b/a$ as at airship. Dimensions of equivalent ellipsoid of revolution under known V and λ are defined by expressions:

a) for oblong ellipsoid of revolution – $V = \frac{4}{3}\pi a^2 b = \frac{4}{3}\pi\lambda a^3;\ a = \sqrt[3]{\frac{3V}{4\pi\lambda}} = 0.62035 V^{1/3}\lambda^{-1/3};$

$$D_{el} = 2a;\ b = a\lambda; \quad (1.13)$$

b) for oblate ellipsoid of revolution – $V = \frac{4}{3}\pi ab^2 = \frac{4}{3}\pi\lambda^2 a^3;\ a = \sqrt[3]{\frac{3V}{4\pi\lambda^2}} = 0.62035 V^{1/3}\lambda^{-2/3};\quad b = a\lambda;$

$$A = 2a;\ D_{el} = 2b; \quad (1.14)$$

Table 2 shows that values of *D* and *L* are rathe closed to values of D_{max} and *L* for real oblong shaped airships.

Comparison of airships and equivalent ellipsoid of revolution dimensions *Table 2*

#	Airship title	λ	D_{max}, m	D_{el} m	D_{max} / D_{el}	Type of design
1	WDL – 1	3,793	14,5	14,456	1,003	*Flexible*
2	WDL – 2	3,89	18,0	18,549	0,970	*Flexible*
3	WDL – 3	4,0	20,0	21,216	0,943	*Flexible*
4	“Star”	2,507	14,6	14,401	1,014	*Flexible*
5	“Santos-Dumont”	2,561	8,59	8,590	1,000	*Flexible*
6	AD – 500	3,571	14,0	13,999	1,000	*Flexible*
7	B – 10	3,864	12,5	12,118	1,031	*Flexible*
8	B – 12 bis	3,98	11,8	11,769	1,003	*Flexible*
9	DM – 20	4,05	13,4	13,310	1,007	*Flexible*
10	N – 1	5,56	19,07	18,522	1,030	*Semi-Rigid*
11	B – 5	5,108	9,3	9,509	0,978	*Semi-Rigid*
12	B – 6	5,56	18,8	18,524	1,015	*Semi-Rigid*
13	B – 7	5,065	15,4	15,301	1,006	*Semi-Rigid*
14	L – 30	8,248	23,9	23,353	1,023	*Rigid*
15	L – 59	9,477	23,9	23,990	0,996	*Rigid*
16	L – 127	7,757	30,5	29,568	1,031	*Rigid*
17	L – 129	6,01	41,2	39,907	1,032	*Rigid*
18	“Akron”	5,9	40,5	39,053	1,037	*Rigid*
19	SL – 1	7,12	18,4	17,506	1,051	*Rigid*
20	SL – 3	7,727	19,8	20,007	0,990	*Rigid*
21	SL – 20	8,66	22,9	23,115	0,991	*Rigid*
22	SL – 120	8,017	35,3	34,341	1,028	*Rigid*

The ellipsoid of revolution surface area can be calculated according to the following rather simple and exact formulas that give good results under fineness ratio value about λ=2…8:

$$S_{el.on.} = 2\pi ab\left(1.5394 - 0.3398/\lambda\right), \text{ m}^2. \quad (1.15)$$

$$S_{el.ot} = 2\pi ab\left(\lambda + 1.1365\lambda^{-0.573}\right), \text{ m}^2. \quad (1.16)$$

Numeral methods design calculation on the base of method of finite differences or any other known methods can be used for more exact calculations of surface area.

The ratio between surface area S and area of middle cross section S_m has important meaning for aerodynamics calculations. The value of middle cross section area S_m for traditional oblong shaped ellipsoid of revolution can be defined with radius $R_{max}=a$ of middle cross section, that is circle, according to the formula: $S_m = \pi a^2$.

And in case of oblate shaped ellipsoid of revolution according to the formula: $S_m = \pi ab$. Than S/S_m can be defined according to the formula:

a) oblong shaped ellipsoid of revolution $\overline{S} = S_{el} / S_M = 3.0788\lambda + 0.6796;$ (1.17)

b) oblate shaped ellipsoid of revolution $\overline{S} = S_{el} / S_M = 2/\lambda + 2.273\lambda^{-0.573}$. (1.18)

§5. Airship Aerodynamic Characteristics.

Drag of the airship is one of the main aerodynamic characteristics, which defines power of the propulsors that is necessary to ensure required flying speed. The drag P_x is connected with drag coefficient $C_{X\Sigma}$ and other design and aerodynamics peculiarities by a relationship:

$$P_X = c_{x\Sigma}\rho S_M w^2 / 2g, \text{ kg} \quad (1.19)$$

Where;

ρ – air and gas densities, kg/m^3;

S_M – mid-section (frontal area; midship), m^2;

W – flying speed, m/s;

g – free fall acceleration, 9.8 m/s^2.

The expression of power *N,* [h.p.], corresponding to given drag, can be gotten of formula (1.19):

$$N = \frac{c_x \rho S_M w^3}{75.2g}, \text{ [h.p.]} \quad (1.20)$$

According to work [7] drag coefficient streamlined body of revolution C_X can be correlated with friction coefficient of flat plate C_{Xf} and with the body geometrical characteristics:

$S=S/S_M$ *, w*here S and S_M - area of outer surface of body (exposed, or wetted area) and mid-section area. The following formula can be used:

$$C_{X\Sigma} = C_{Xw} + C_{Xb} + C_{Xf}\overline{S} + \Sigma\left(C_{Xss}S_{Mss}\right)/S_M, \quad 1.21)$$

where:

C_{Xw} – wave-drag coefficient, needs to be taken into consideration under super sonic flying speed;

C_{Xb} – base drag coefficient, needs to be taken into consideration for bodies with flat bottom at the tail part;

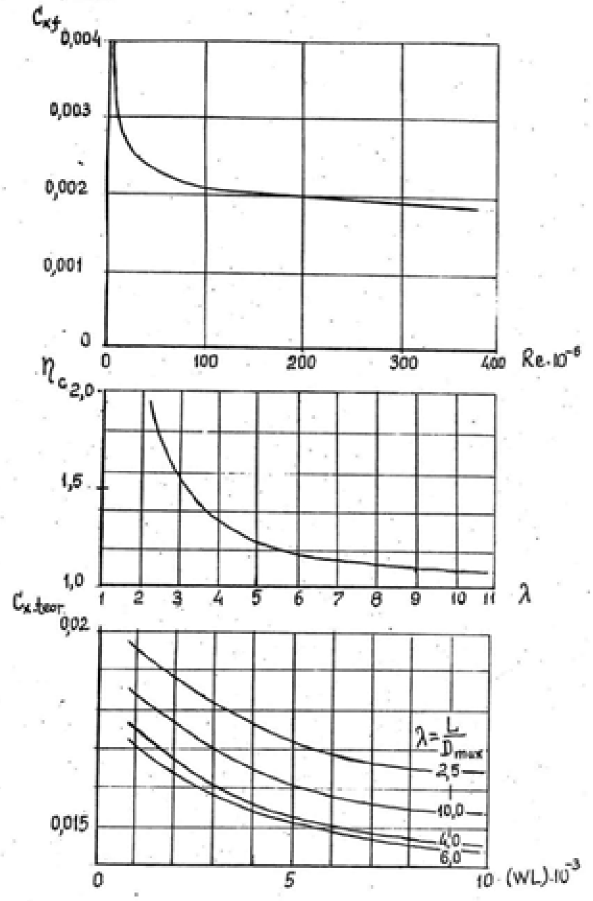
η_c – the transition coefficient from flat plate to the body of revolution;

$C_{Xss} \cdot S_{Mss}$ – drag coefficient of superstructure and mid-section area of superstructure.

The values of C_{Xf} and η_c coefficients in dependence of fineness ratio (for oblong streamlined body of revolution) are shown in Fig.7. As it is obvious in the equation (1.21) its first and second members can be ignored in case of airships due to flying speed is under 100…200 km/h and tail part of the airship body is streamlined.

Results of $C_{Xteor.}=C_{Xf}\eta_c\overline{S}$ calculations are shown in Fig.7.

Fig.7. Coefficients $C_{X\,f}$, η_C and $C_{X\,teor}$ changing according to Re, λ and (WL)



Thus, increasing of *Re* number (and proportional to it value WL) leads to stabilizing value of C_{Xteor} due to self-similarity phenomena under big magnitudes of *Re* number. Furthermore there is minimum of C_X magnitude under λ = 5…6.

Nevertheless, absolute magnitude of C_{Xteor} is much less than real magnitude of $C_{X\Sigma}$ that is typical for airships. It may be grounded by additional losses on friction, vortex generation, caused by design features of envelope, its vibration in flight, tail unit, rudders and elevators, gondolas and other factors influence, that is difficult to take into account. In it, it's in point to remark that according to data of [7] the flush riveting of metal skin increases C_X by 0.00015…0.00020. Even two rows of rivets heads on aircraft skin increases C_X by 0.020…0.025.

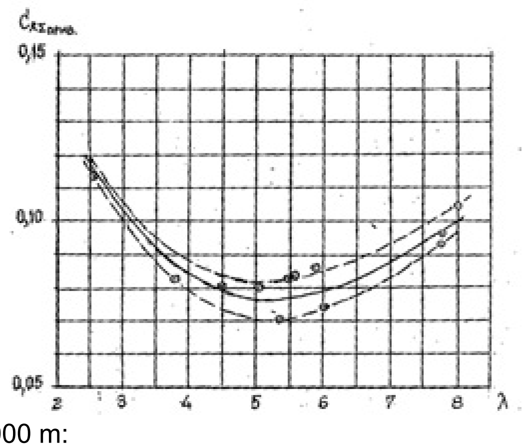
The value $\Sigma(C_{Xss}\ S_{ss})/S_M$ for airship must not increase total drag coefficient significantly due to its very small part in body drag. That is why investigation of total drag coefficient including all losses connected with body and its additional and auxiliary superstructures of concrete airship design conception is more reasonable. In this case analysis of airships statistical data is more preferable, because now there are no summarized theoretical equations. Statistical data of real airships give equations that can be used for definition of realistic $C_{X\Sigma}$. There is every reason to suppose that common logic for ideal smooth body shown in Fig.7 is true for airship body, but on another quantitative level and they can be used as a basic mathematical model for treatment of airships statistics. $C_{X\Sigma}$ were calculated by formula (1.20) on the base of data submitted in [1,2] about most perfect airships from aerodynamic point of view.

Power *N* being a member of formula (1.20), according to statistics presents the full (gross) power of engines. This value is more than power of propulsors needing to overpower the drag N_T. Both values *N* and N_T are connected with one another by the ratio:

$$N = N_{teor.} / \eta_{pr}\text{, h.p.,} \qquad (1.22)$$

where: η_{pr} – propeller efficiency, its average meaning is 0.80…0.85.

The value C_X, calculated with formula (1.20) on the base of statistics for *N*, will be more than value calculated for power of propellers by value $1/\eta_B$, but in this case we get shaft power of engines, and thus, it is possible to ignore the concrete characteristics of the propellers.



On the base of correlation analysis, and taking into account qualitative dependence in Fig. 6,7 for streamlined bodies of revolution, ratio for C_x reduced to fixed magnitude *WL* = 10000 (*Re*=6.86x10^8) under *H*=0, (Fig.8)

Fig.8. $C_{x\ red}$ -to- λ dependence for real airships (reduced to Re=6.86x10^8).

was received by the treatment of existing data. The dependence for $C_{X\Sigma}$ was received as a subject to influence *WL* and λ for airships of classic shape under altitude below 1000 m:

$$C_{X\Sigma} = \left[0.5354 + 0.0305 \cdot (\lambda - 5.55)^2\right] \cdot (WL)^{-0.21}, \qquad (1.23)$$

The universal formula to define the $C_{X\Sigma}$ of classic and untraditional shape can be derived by using expressions like $\bar{S}=S_{ell}/S_M$ for λ (1.17) and substituting $Re \cdot v$ for WL. The formula can be used for large scale of geometric rations, flying speeds and altitudes:

$$C_{X\Sigma}=\left[5.54134 + 0.3162\left(0.3248\bar{S} - 5.7707\right)^2\right]\mathrm{Re}^{-0.21}. \qquad (1.24)$$

The comparison of real and calculated data according to the formula (Table 3) shows its rather good correlation in the limits of exactness of initial data.

According to prognosis [1], the value of $C_{X\Sigma}$ can be decreased by 10% for perspective airships in comparison with achieved value of $C_{X\Sigma}$.

The comparison of real and calculated values of $C_{X\Sigma}$ *Table 3*

NN	Airship	λ	L, m	N, h.p.	W, m/s	$C_{X\Sigma}$	$C_{X\Sigma calc.}$	$C_{X\Sigma calc.}/C_{X\Sigma}$
1	R-101	5.49	219.5	2925	31.39	0.0904	0.08356	0.9244
2	R-100	5.33	216.5	3600	36.67	0.0745	0.08130	1.0915
3	N-1	5.56	106.0	750	31.39	0.1020	0.09523	0.9336
4	"Santos Dumont"	2.56	22.0	40	15.28	0.2324	0.23780	1.0234
5	WDL-1	3.79	55.0	360	27.78	0.1222	0.13480	1.1032
6	B-1	4.50	45.0	150	26.39	0.1248	0.12850	1.0293
7	B-5	5.56	104.5	810	31.39	0.1133	0.09764	0.8617
8	B-7	5.06	78.0	730	35.56	0.1047	0.10250	0.9790
9	SL-Atl.2	7.78	266.0	3500	36.11	0.0972	0.09998	1.0286
10	SL-120	8.02	283.0	4000	36.11	0.1043	0.10360	0.9934
11	LZ-100	8.24	197.0	1450	31.67	0.1212	0.12053	0.9880
12	LZ-126	7.25	200.0	2000	35.00	0.0940	0.09700	1.0319
13	LZ-127	7.76	236.6	2650	35.56	0.0969	0.10239	1.0566
14	LZ-129	6.01	247.8	4400	37.50	0.0752	0.07940	1.0559
15	"Akron"	5.90	239.0	4480	36.11	0.0887	0.08024	0.9047

§6. Airships Weight Characteristics.

Weight characteristics are very important for aerostatic vehicle, as far as they define payload and optimal parameters in dependence on its prescript aim function. It will be reasonable to submit total weight of airship as balanced by lift of carrier gas in form of a blocks row. The sum of blocks weight will be different in dependence of different combinations of design factors.

The following special blocks can be defined for airships of different type of design scheme (flexible, semi-rigid and rigid):

1. Envelope and gas holder skin. Area of its surfaces and characteristics of skin material define the block weight.
2. The rigid framework (for semi-rigid and rigid airships).
3. Engines and propellers weight, defined in dependence of altitude and speed that are basis data for calculation of design parameters and engines power.
4. Useful load including fuel for engines. Obviously, that last article including compartments for crew, passengers, cargo and cargo itself will depend on difference between buoyant lift and sum of first three blocks weight. Correlation between fuel weight and other shares of commercial load will define the flying range.

The weight of first block defines mainly by the weight of 1 m^2 envelope skin and by weight of gas holders, that can be defined from data of weight characteristics of materials used in concrete structures. The value can be approved as 0.5 kg/m^2 of envelope surface for existed rigid dirigibles that was calculated on the base of statistics data analysis.

The weight of second block, typical for rigid and semi-rigid vehicles, depends on design conception, structure materials and its volume and its design relationships.

The results of weight framework analysis under different design relationships, in according with loads acting to the body, can be used for evaluation of dependence defining the weight of the second block.

Weight of engines and propulsors can be defined on the base of design characteristics of the engines and scheme of used propeller drive.

Thereby, the used in shipbuilding concept of "dead weight", is reasonable to investigate for different type of airship design. Here the "dead weight" means a difference between buoyancy of lifting gas at the altitude

H=0 and weight of framework with envelope. The obtained difference between mentioned weights could be used in the diverse ways. It depends on airship purpose, or in another words, on its aim function. The most part of dead weight is used for fuel in case of long range airship and for short range airship the most part of dead weight (including weight of the engines) could be used for maximum payload. The calculation of rigid airship framework weight is most difficult problem due to arrangement of a wide ambit of airship volumes and load-carrying capacity. Weights of airframe depend of their strength and stiffness. Evaluation of the characteristics is rather difficult on the step of draft design due to absence of concrete generalized recommendations. That is why it is reasonable to set defined criteria connected with the strength and stiffness of rigid airship carried framework. The criteria allow more exactly define the framework weight at the initial stage of draft design. The mechanism of acting to framework forces and their influence to the airship framework construction weight need to be analyzed for these purposes. A distribution of specific gas load on a unit of airship body length, created by lifting gas buoyancy, and concentrated loads acting on ellipsoid shape airship is shown in Fig.9.

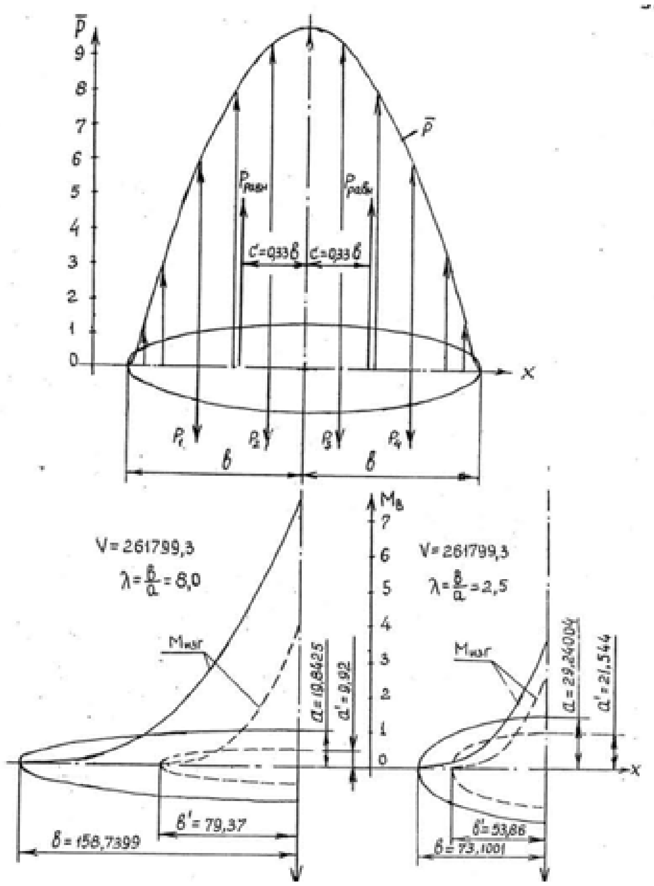


Fig.9. Relative specific gas load distribution P and bending moments M_{bend} along the ellipsoid shape airships length:
----------- oblong shape and - - - - - - - oblate shape ellipsoids of revolution.

Generally, in order to simplify this analysis, all concentrated loads can be transferred to one concentrated load applied in the center of the airship body. Buoyancy creates bending moments diagram along the airship body length. The value of bending moment is variable and its maximum is located in the middle of the airship body length. In this case danger section is middle one. As analysis shows, a resultant force of buoyancy for every symmetrical half part of the body is applied at one-third of half length of the ellipsoid *l`=b/3* from its middle section.

Diagrams of gas load distribution in dimensionless coordinates $\bar{x} = x_i / b$ and P/P_{max} have the same form both for oblong and oblate shape ellipsoids of revolution as well. The fact confirms similarity of diagrams bending moments of such ellipsoids with any between semi-axis *a* to semi-axis *b* relations. The maximum bending moment M_{max} depends on length of ellipsoid of revolution only under its given volume *V*, due to resultant gas lift stays the same, and shoulder of acting force is one-third of semi-axis of generating ellipse as well.

In case of ellipsoid skin with thin ring shape cross-section (Fig.10)

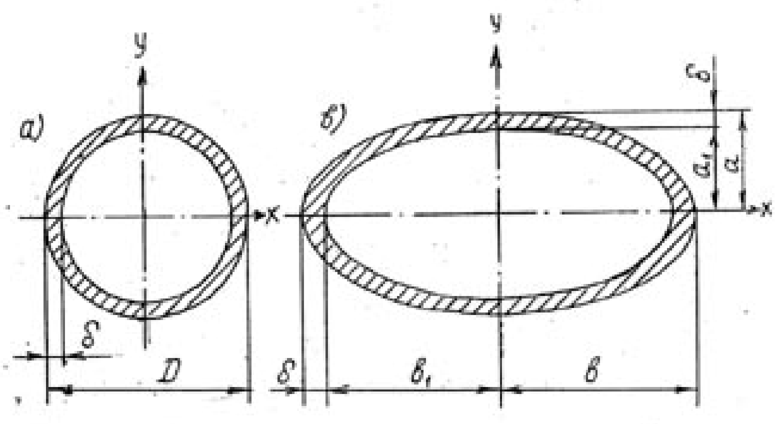


Fig.10. *Conventional cross-section of the rigid body ellipsoid shape airship: a) – oblong; b) – oblate.*

under condition of isorugged structure of framework, the thickness of the skin δ_i can be defined according to the formula of thin ring section modulus W_i:

$$W_i = \frac{\pi D_i^2 \delta_i}{4}; \quad \delta_i = \frac{4W_i}{\pi D_i^2}; \tag{1.25}$$

In case of isorugged structure (bending stresses $\sigma = const$) for airship body of minimum weight, the volume of section modulus in every cross-section is defined by value of bending momentum in the same cross-section:

$$\sigma = \frac{M_i}{W_i} = const. \tag{1.26}$$

Maximum thickness of the rigid airship skin is located in the middle cross-section. Due to bending moments diagrams are geometrically similar under different cases, then thickness of the skin is changed according the same principle. This circumstance allows to formulate the mathematical model of relative framework weight change with given volume under different fineness ratio $\lambda = L/D_{i\,max}$. Ratio of construction weights G_i and G_0 of two airship frameworks and different fineness ratio λ_i and λ_0 under the same given value V and $\sigma = const$ is proportional to the ratio of the products ring thickness δ_i in maximum cross-section, length of body Li and maximum diameter D_i, since the element of value $\Delta V = \pi D\delta\Delta l$, and volume δ_i varies in inverse proportion to square of diameter D_i or directly proportional to square of length L_i:

$$\frac{G_i}{G_0} = \frac{\delta_i L_i D_i}{\delta_0 L_0 D_0} = \left(\frac{L_i}{L_0}\right)^3 \cdot \left(\frac{D_i}{D_0}\right) = \left(\frac{\lambda_i}{\lambda_0}\right)^3 \cdot \left(\frac{D_i}{D_0}\right)^4. \tag{1.27}$$

Since the volume of oblong ellipsoid of revolution equals to:

$$V_{el} = \frac{4}{3}\pi\left(\frac{L}{2}\right)\left(\frac{D}{2}\right)^2 = \frac{4}{3}\pi\lambda\left(\frac{D}{2}\right)^3, \tag{1.28}$$

then ratio is:

$$\left(\frac{D_i}{D_0}\right)^4 = \left(\frac{\lambda_i}{\lambda_0}\right)^{4/3}. \tag{1.29}$$

Hence it is possible to get relation between weights of framework constructions of two airships with different fineness ratio λ and the same volume under equal strength σ in framework elements:

$$\frac{G_i}{G_0} = \left(\frac{\lambda_i}{\lambda_0}\right)^{5/3}. \tag{1.30}$$

The same method can be used during investigation of relations between weight of two airships having equal fineness ratio and different volume. It is possible to prove that framework weight changes proportionally to forth rate of diameter or to value $V^{4/3}$.

Summarizing the relations between weights of airships with different volumes and fineness ratio, it is possible to deduce a general formula of volume V and fineness ratio λ influence to rigid type airship framework weight:

$$\frac{G_i}{G_0}=\left(\frac{V_i}{V_0}\right)^{4/3}\left(\frac{\lambda_i}{\lambda_0}\right)^{5/3}. \tag{1.31}$$

Disposing of reliable data about weights of appointed structure principle with different volumes and fineness ratios and using the formula (1.31), it stays possible to get a dependence that can evaluate exact weight of rigid framework structure G_K of oblong ellipsoid of revolution type:

$$G_K = A_{on}V^{4/3}\lambda^{5/3}\text{ , kg} \tag{1.32}$$

Coefficient A_{on} characterizes oblong shape airship of fixed design principle made of suitable materials and reflects the achieved level of perfection. As far as this technique sphere is developing, the value A_{ol} has to be decreased, demonstrating real progress in framework weight reduction.
The weight of oblate ellipsoid shape airship (disk shape form) can be expressed by formula:

$$G_K = A_{ot}V^{4/3}\lambda^{1.239}\text{, kg} \tag{1.33}$$

Coefficients A_{ol} and A_{ot} can be called as an *airship weight criteria* for oblong and oblate shape airships accordingly. It will be shown later that they characterize the level of strength and deformations of rigid framework as well.
Considering the attained value of criteria A_B for the rigid Zeppelins (Table 4), we can notice, that this value in earliest designs (LZ-2, LZ-3, LZ-4) was about (5…6)·10^{-4}, and for airships LZ-59, LZ-127 it reached its minimum of 3·10^{-4}, but in last generation airships (LZ-129, LZ-130, "Akron") it increased up to 5·10^{-4} anew.
Hence it follows a conclusion, that *minimum value of* $A_B=3\cdot10^{-4}$ *allows to obtain an airship of minimum weight and maximum dead-weight* under constant strain deformation $\bar{y}$ =*y/L*, where *y*- absolute deformation of framework.

Criteria of weight, absolute and related sagging of a stiff airframe for the rigid Zeppelin's airships. *Table 4*

NN	*Airship*	*Year*	*V, m³*	*λ*	L=2b, m	S_{ell}, m²	G_{en}, kg	G_K, kg	$A_B\cdot10^4$	$\bar{y}_B\cdot10^9$	$(\bar{y}_B/b)\cdot10^7$
1	LZ-2	1905	11 300	11,034	128,00	4 256,2	2 128,1	7 665,2	5,527	3,167	4,949
2	LZ-3	1905	11 397	10,940	128,00	4 269,1	2 134,6	7 244,8	5,239	3,303	5,161
3	LZ-4	1908	15 000	10,462	136,00	5 055,8	2 527,9	9 537,1	5,152	3,417	5,025
4	L-30	1916	55 000	8,284	198,00	11 182,3	5 591,2	26 663,9	3,759	4,895	4,945
5	L-59	1917	68 500	9,477	226,50	13 493,9	6 747,0	33 611,4	2,826	8,766	7,741
6	LZ-127	1928	105 000	7,757	236,60	16 865,8	8 432,9	47 397,1	3,148	6,499	5,494
7	LZ-129	1936	190 000	6,015	247,80	23 999,7	11 999,6	110 593,5	5,089	3,206	2,587
8	"Akron"	1931	184 000	5,900	239,00	22 572,0	11 286,0	93 971,6	4,661	3,353	2,806

But this conclusion leads to paradox results, namely under small volume V, weight of framework G_K becomes very small according to scale similarity *V→0,* and this is quite unreal think. Under big volume (*V* till 200 000 m³ and more accordantly) constancy of A_{ot}, being accomplished by constant deformation $\bar{y}$*=y/L,* results to very big *y*. Thus, the admissible stiffness demand begins to predominate over the strength demand, so it leads to necessity to increase the weight of framework under big airship volume *V.*
As a result the conception *A=const* can not be applied to airship designing. Analysis of existed designs of the airships show that as one of the main design and weight characteristics of airships can be choose the weight of framework related to the envelope surface*:* $g_S= G_K/S_{el}$, kg/m².
Employing the expressions for framework weight (1.32) and expression for surface square of oblong shape ellipsoid in transformed view:

$$S_{el}=2.4179V^{2/3}\lambda^{1/3}\cdot\left(1.5394+\frac{0.3398}{\lambda}\right),\text{ m}^2 \tag{1.34}$$

one can to deduce an expression for g_S*:*

$$g_S=\frac{A_{on}V^{2/3}\lambda^{4/3}}{2.4179\cdot(1.5394+0.3398/\lambda)}. \tag{1.35}$$

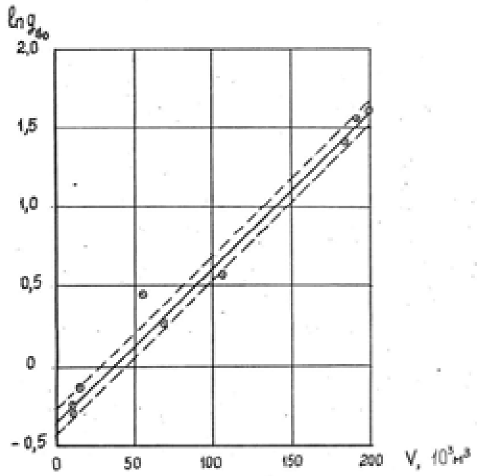
The value g_S can be recalculated for airships with different framework *λ* by reduced formula under defined *λ* and *V=const; A =const*:

$$\frac{g_{S1}}{g_{S2}} = \left(\frac{\lambda_1}{\lambda_2}\right)^{4/3} \cdot \frac{(1.5394 + 0.3398/\lambda_2)}{(1.5394 + 0.3398/\lambda_1)}. \qquad (1.36)$$

The weight of rigid framework G_K and weight of the envelope with gas holders G_{ob} need to be investigated separately to deduce a relations, which defines design principle, processing from a characteristic *g*=G/S=0.5 kg/m^2. The value 0.5 kg/m^2 is accounted as specific weight characteristics, applied for construction of Zeppelins' envelope and gas holders. Variation of this value within the limits *g*=(0,5…1,0) kg/ m^2 does not lead to considerable error in a general relation and it influences very little on final results.

Reducing value g_s to the fineness ratio *λ* =6,0, one can to get dependence *g=f(V)* for Zeppelin's airships (see Fig.11).

Fig.11. V - to - ln g_s dependence (transformed to λ =6,0).



The dependence is good approximated by exponential curve:

$$g_{S0} = \exp\left(9.75 \cdot 10^{-6} V - 0.35\right), \text{ kg/m}^2 \qquad (1.37)$$

General expressions for A_{ol} and g_s can be used for equation of construction Zeppelin's conception that shows changing of weight criteria A_{ol} on structural parameters of airships:

$$A_{on} = \frac{G_k}{V^{4/3}\lambda^{5/3}} = \frac{S_{el} g_s}{V^{4/3}\lambda^{5/3}} = 0.354 V^{-2/3} \exp\left(9.75 \cdot 10^{-6} V - 0.35\right). \qquad (1.38)$$

A comparison of points calculated according to equation of construction conception with real data (Fig. 12) shows their good convergence.

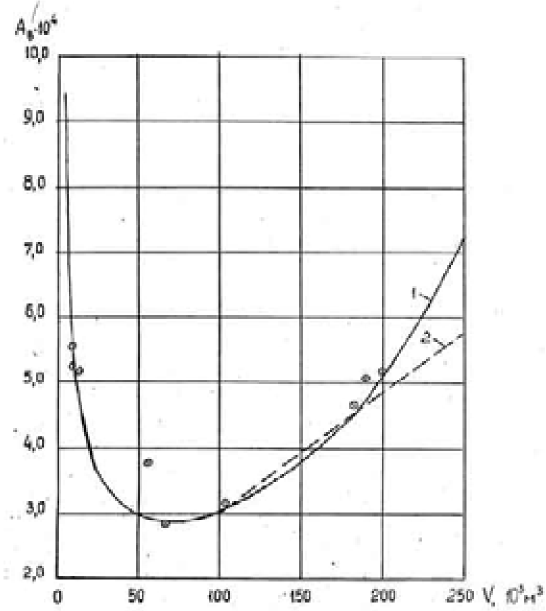


Fig. 12. Weight criteria A_{ol} of value V dependence: line according to construction Zeppelin's conception.

Taking into consideration absence of real construction of such type airship with value above 185000 m^3 and evaluating the exponential extrapolation as rather risky under value in some times more of reached real maximum value, the linear dependence was chosen for calculation:

$$A_{on} = 1.7 \cdot 10^{-9} V + 1.5 \cdot 10^{-4}. \qquad (1.39)$$

The linear dependence is preferable for calculation until new real and grounded data will be received.

§7. Airship Framework Strength Criteria.

Considering simplified scheme of load acting on load-bearing framework of isorugged structure *(σ = const*) of rigid airship (Fig.9) it is possible to define bending stresses in any cross-section by formula:

$$\sigma = \frac{M}{W_X} = \frac{P_B L}{2 \cdot 6 W_X} = \frac{\Delta\rho}{2} \cdot \frac{L}{6W_X} = \frac{\Delta\rho V L}{12 W_X}. \text{ kg/m}^2 \quad (1.40)$$

where: $\Delta\rho$ – specific buoyancy of 1 m ^{3} of gas (for helium under H=0 m, $\Delta\rho$ = 1,0564 kg/m^3);
P_B – total buoyant lift of gas into the envelope, kg;
L – airship length, m;
V – volume of the envelope of the airship, m^3;
W_x – moment of resistance against bending, relatively of axis X-X (Fig.10).
Values of resistance moments W_x to bending in the mid-section relatively of axis X-X for oblong and oblate shape ellipsoids of revolution accordingly equal to (see Fig.10):

$$W_{Xon} = \frac{\pi D_{\max}^2 \delta_{\max}}{4}; \text{ m}^3. \quad W_{Xot} = \frac{\pi}{4} \cdot \left(\frac{ba^3 - b_1 a_1^3}{a} \right); \text{ m}^3 \quad (1.41)$$

The simple formula of rigid airship construction weight definition was set up on the base of isorugged structure of oblong shape and oblate shape ellipsoids of revolution framework analysis:

$$G_S = F_{\max} \rho L / 3, \text{ kg}, \quad (1.42),$$

where F_{max} – frontal area of conventional envelope, that is equivalent to weight of airship framework, m^2.
ρ – density of the airship framework structure material (for aluminum alloys ρ = 2780kg/ m^3);
The formula (1.42) is transferred in case of oblong shape ellipsoid of revolution and aluminum alloy into expression:

$$G_S = \pi D_{\max} 2780 L / 3 \text{ , kg}, \quad (1.43)$$

where δ- thickness of conventional skin, m;
D_{max} –mid-section diameter, m;
Using the expression (1.32) for the framework weight and taking into consideration that volume of oblong shape ellipsoid of revolution V_{el} equals to:

$$V_{el} = \frac{4}{3} \pi a^2 b, \text{ m}^3, \quad (1.44)$$

where $b = L/2$; $a = D_{max}/2$; thus it is possible to get:

$$\sigma = C_1 A_{on} V^{2/3} \lambda^{4/3} \text{ , m}, \quad (1.45)$$

where $C_1 = 3/[\pi (6/\pi)^{2/3} \rho] = 2.2315 \cdot 10^{-4}$ with ρ = 2780 kg/m^3.
Taking into account expressions for δ_i and W_i (1.25) bending stress σ in the case equals:

$$\sigma = C_1 / A_{on} \quad (1.46)$$

Expression for calculation of σ in case of oblate shape ellipsoids of revolution can be received analogically. Therefore the product $A_{on}\sigma$ = *const* = 404,853 (for oblong) and $A_{ot} \cdot \sigma$ = *const* = 353,477 (for oblate) shape ellipsoids with any fineness ratio λ and any volume V. One can see, that criteria A characterizes as a weight as a stress level in rigid framework as well.
It is obvious clear from the mentioned above equations, that oblate airship framework is less than weight of oblong one under all other equal conditions and equal stresses in rigid framework and value of coefficient A_{on} is defined by formula:

$$A_{ot} = A_{on} \frac{353.477}{404.853} = 0.8731 A_{on}. \quad (1.47)$$

§8. Airship Framework Stiffness Criteria.

Conventional values of absolute deflection and related deflection $\bar{y} = y / b$ can be used as framework stiffness criteria for the scheme of above mentioned airship framework loading. The value of framework deflection y for conventional scheme with constant moment of inertia of cross-section $J_{X\ max}$ along the framework is defined by the expression:

$$y_{\max} = \frac{4Pb^3}{3BI_X} = \frac{4\Delta\rho V L^3}{3E \cdot 2.8} \text{ , m}. \quad (1.48)$$

The value of moment of inertia for circular cross-section (oblong shape ellipsoid of revolution) equals to:

$$I_{Xon} = \frac{\pi D_{\max}^3 \delta}{8}, \text{ m}^4. \quad (1.49)$$

For the elliptical cross-section (oblate shape ellipsoid of revolution) the value J_X equals to (see Fig.10):

$$I_{Xot} = \frac{\pi}{4}\left(ba^3 - b_1 a_1^3\right), \text{ m}^4. \quad (1.50)$$

The value J_{XB} for oblong shape ellipsoid of revolution has being substituted into the expression for Y_{max}, can get criteria expression, considering maximum deformation of framework structure after discarding of some constants in it:

$$y_{\max} = \frac{4\Delta\rho V \lambda^3}{6E\pi\sigma} = a_1 \frac{V\lambda^3}{\pi\delta} = a_2 \frac{V^{1/3}\lambda^{5/3}}{A_{on}}; \quad (1.51)$$

$$y'_B = \frac{y_{\max}}{C_1} = 1426.452 V^{1/3} \lambda^{5/3} A_{on}^{-1}.$$

Criteria of absolute deflection for oblate shape ellipsoid of revolution can be got in the same way:

$$y'_C = 1426.452 V^{1/3} \lambda^{4/3} A_{on}^{-1}. \quad (1.52)$$

Criteria of related deflection $\overline{y} = y/b$ both for oblong and oblate shape after discarding of some constant coefficients gains the same view:

$$\overline{y}_{on} = \lambda / A_{ot}. \quad (1.53)$$

Adduced expressions show that criteria A under defined value of fineness ratio λ determines the equal related deformations of airship framework. The expressions (1.52) and (1.53) show, that due to absolute framework deformations of oblate shape are less than ones of oblong shape and with other equal conditions, despite equal related deformations $\overline{y}$, the stiffness of oblate shape framework is higher than oblate shape framework. This circumstance can allow to increase acting stresses σ and to decrease the weight of oblate shape framework.

The improvement of design, utilization of new materials and technologies will make for deviation from Zeppelin design conception to the direction of weight criteria *A* decreasing, but the general logic will remain valid according to the strength and stiffness requirements.

§9. Maximum and Optimal Parameters of Gas-filled Airships Designs.

Dependence (1.32) and (1.33) define the weight of rigid airship framework, as it was shown in previous chapters. Taking into account that weight criteria A_{ol} and A_{on} are functions of airship and using formulae (1.38) and (1.39), the following expressions for oblong shape ellipsoid of revolution framework can be received:

with V‹185000 m³: $G_{fw1}^{on} = 0.354 V^{+2/3} \lambda^{5/3} \exp\left(9.75 \cdot 10^{-6} V - 0.35\right), kg$ (1.54)

with V›185000 m³: $G_{fw2}^{on} = \left(1.7 \cdot 10^{-9} V + 1.5 \cdot 10^{-4}\right) \cdot V^{4/3} \lambda^{5/3}, kg$, (1.55)

The weights of oblate ones according to expressions (1.33) and (1.47) are defined by the formulae:

with V‹185000 m³: $G_{fw1}^{ot} = 0.3091 \cdot V^{2/3} \lambda^{1.239} \exp\left(9.75 \cdot 10^{-6} V - 0.35\right), kg$ (1.56)

with V›185000 m³: $G_{fw2}^{ot} = 0.8731 \cdot \left(1.7 \cdot 10^{-9} V + 1.5 \cdot 10^{-4}\right) \cdot V^{4/3} \lambda^{1.239}, kg$, (1.57)

Examining the summary weight of construction G_Σ, (that includes framework weight G_K plus the envelope and gas holders weights G_{ob},) one can observe that for every fineness ratio *λ =L/D =b/a,* there are two limit volumes V_{min} and V_{max}, when summary weight of structure equals to buoyant lift of gas; it means that deadweight $D_w = 0$.

Maximum volume V_{max} corresponding to $D_w = 0$ has no any practical sense; since it is necessary to define the volume *V* corresponding to maximum deadweight D_w under best economical and technical performance of the airship. This volume can be found by methods of function extremum definition, defining summary weight G_Σ:

$$G_\Sigma = G_S + G_{en} = AV^{4/3}\lambda^n + S_{el}g_{en}, kg \qquad (1.58)$$

$$D_W = P_b - G_\Sigma = V\Delta\rho - G_\Sigma, kg\ , \qquad (1.59)$$

where S_{el} –square of equivalent ellipsoid surface (formulas 1.15 and 1.16), m^2;
G_{en} – weight of the envelope and gas holders per 1 m^2 of square surface S_{el}, kg/m^2;
n=5/3 and =1.239 for oblong and oblate shapes accordingly;
P_b – buoyant lift, kg.
Minimum limit volume V_{min} has a practical sense from the point of view of making the experimental structures with minimum size to improve design decisions, technology of manufacturing, making of unmanned and manned airships for special purposes.
The volume of airships were calculated for different fineness ratio λ, corresponding to D_w=0 (V_{min}) and $D_{w\,max}$, under the value g_{ob} =0.5kg/m^2 and for perspective constructions as well, on the base of deadweight expression (1.59) with use of Zeppelin's construction conception evaluations (1.38) (1.39) and expressions for G_Σ (1.54) - (1.57).
According to existing forecast [1] there is reason to hope that weight of envelope and gas holders can be decreased by two times. The weight of rigid framework can be decreased by 25 % in comparison with airships of past.

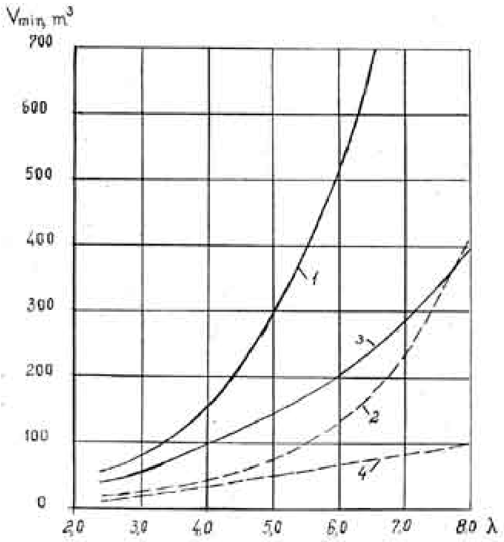
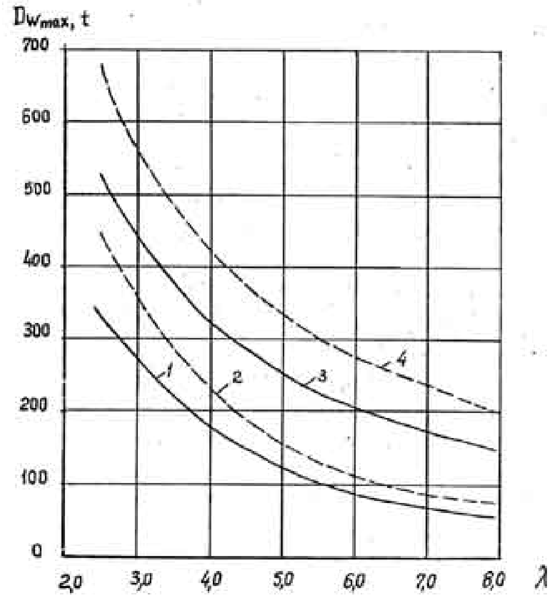


Fig.13. Airship minimum volume V_{min} to fineness ratio λ dependence:
1 - oblong shape airship (Zeppelin 's conception), 2 – the same for perspective constructions;
3 - oblate shape airship (Zeppelin 's conception); 4 - the same for perspective constructions.

The following values were accepted for perspective design of airships: g_{ob} = 0.25kg/m^2, $\mathbf{G_K}$ = 0.75G_{KO},
where G_{KO} - defined according to Zeppelin's conception framework weight. Results of calculations are shown at Fig. 13 and Fig.14.

Fig.13 demonstrates the volumes V_{min} with D_w=0.

Fig.14. Dependence of maximum deadweight D_{Wmax} from fineness ratio λ:
1- oblong shape airship (Zeppelin 's conception); 2 – the same for perspective constructions;
3 - oblate shape airship (Zeppelin 's conception); 4 - the same for perspective constructions.



The dependence of maximum deadweight D_w of λ for perspective airships corresponding to Zeppelin's conception is shown on Fig.14. One can see, that maximum deadweight under minimum fineness ratio λ =2.5 is under 700 t for perspective oblate shape airship and 450 t for oblong shape airship. For Zeppelin's conception airships these values equal accordingly 530 and 330t. An increasing of λ leads to dramatic dropping of deadweight; so, with λ = 8 D_w =200t and 75 t for perspective airships and D_w =145t and 55 t for Zeppelin's conception airships. Payload is less than deadweight on value of engines and fuel

weight in dependence of altitude, range and flying speed. Considering received deadweight limits of different airships designs, statement of an optimization problem of airship main design and operational parameters in dependence on exact aim function is reasonable.
Optimization airships parameters under maximum deadweight $D_{W\,max}$ can be investigated as an example.
Let's suppose the following parameters need to be defined: volume V, fineness ratio λ and flying speed at given altitude under defined range L_M, km (scheduled cargo route) under different optimization criteria. Optimization criteria can be defined on the base of different approaches. Let us assume that in first case transportation of maximum weight cargo is wanted for distance L_M =2000 km at the altitude H=0, and other factors such as: transportation endurance, fuel consumption and other expenses are not very important and can be ignored. The maximum load capacity is the main optimization criteria in this case and as it was calculated (Fig.15) for this optimization criteria the oblate shape airship with minimum fineness ratio λ =2.5 is the best under minimum flying speed W_{min} =100km/h.

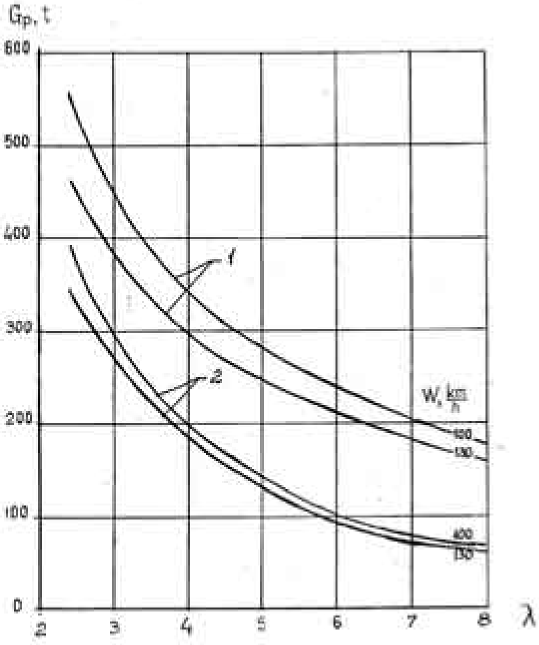


Fig.15. Cargo capacity C_{ar} of gas airship under maximal deadweight, H=0, L_M=2000km

1 – of oblate shape; 2 - of oblong shape

Cargo capacity C_{ar} is 539.997t under the mentioned condition. Fuel consumption will be about 129.093 t per one flight. Specific fuel consumption is g_f = 0.1195 kg/t·km. The cargo turnover is more important characteristic, than g_f under scheduled cargo operation, or in other words the amount of cargo transported per time unit. Increasing of flights number n_{flt} per day needs to increase the speed of flight. W and consequently it will be bounded up with building up of engines power, fuel capacity and decreasing of cargo capacity. The optimum choice for the case is the oblate shape airship with λ =2.5 and flying speed W=130 km/h. The maximum cargo turnover will be G_{car} n_{flt}=699.949 t/day under cargo capacity G_{car}=448.68 t. About 206.473t of fuel will be used up, and specific fuel consumption for one ton-kilometer will be g_f = 0.23 kg/t·km.
However such approach may be not very satisfactorily for optimization of transportation vehicles used on the cargo line. Profitableness of the airship can be reached under minimum value of general effect criteria – a sum of annual total expenses related to one ton-kilometer of transportation.
Annual expenses includes the following:

1. Amortization charges C_a that are equal to capital expenses divided to life time of the airship and related to one ton-kilometer of transportation;
2. Operational expenses. Salary is the biggest part of it and maintenance, plus repair, refueling with the fuel or gas and another works such as ground infrastructure for airship service;
3. Fuel expenses depending of engine power and time of their operation.

Definition of every above mentioned point is rather difficult problem, for exception of last one. Resolve of every this problem became available only after accumulation of airship use experience, and its manufacturing, and maintenance of investigated type vehicles as well.
In order to work out above-mentioned problems it is reasonable to estimate the shares of this points of annual expenses in total sum on the base of modern conception as to the air transport.
Fuel consumption definition or fuel efficiency that can be expressed in value of consumed fuel per 1 t·km is not difficult task due to shown in previous chapters dependence, which can be used for the resolve. The final expression can be presented as a simple formula:

$$\bar{g} = \frac{G_{f.h} \cdot 24}{G_P n_{flt} L_M}, \text{ kg/t·km,} \qquad (1.60)$$

where $G_{f.h}$ – hourly fuel consumption, kg/h.

According to aviation statistics the share of this expenses can reach from 10 till 30 % of total annual expenses sum. The largest part of expenses is operational costs C_i. This part can account for 50 till 80% of total expenses, as the salary is from 30 till 60% of total operational costs. The sum of pilots and technicians salary, costs for refueling of lifting gas, for maintenance and repair – all of these are approximately constant values and they do not depend on number of flights and value of transported cargo. That is why annual maintenance expenses can be evaluated by some constant, that characterizes the aircraft, divided on cargo turnover under constant range (leg) of flight L_m. The airship volume *V* can be approved as a constant, that effects on mentioned above elements of maintenance cost more than other factors: number of crew, ground team, value of lifting gas leakage and other elements depending on airship volume *V.* In this case as a conventional characteristic of annual operational expenses referred to one ton-kilometer can be the quotient of *V* and quantity of ton-kilometers, transported per time unit (it does not matter which one – it can be days, for example):

$$C_e = \frac{V}{G_P n_{flt} L_V}. \qquad (1.61)$$

To draw a comparison between these values of different airships peculiarities of structures and performance it is possible to get relative deviation of value C_{ei}.

Amortization *Ca* connected with the cost of airship is difficult to define as previous value as well, and its characterizing value can be estimated as proportional cost-to-weight relation. In that case the value *Ca* calculates by expression:

$$C_a = \frac{G_\Sigma}{G_P n_{flt} L_V}. \qquad (1.62)$$

The part of amortization in total sum referred to annual expenses is about (5…20)%, and in any case this value is less than expenses for refueling C_f.

To obtain relative variation of every point of expenses and total costs $\Sigma C_i = C_f + C_e + C_a$ for some options, we assume percentage relations between C_f, C_e and C_a as a base values for arbitrary airship design variant. In this case it is possible to find relative variation of expenses point as a quantity of percents comparing with basics value ΣC_i=100%.

Exact magnitude of C_f, (kg/t·km) for every variant under known cost of fuel gives possibility to estimate other points of expenses quantitatively.

In given case for basics variant maximum likelihood relations between shares of expenses are applied: C_f =20%; C_e=70%; C_a=10%. Deviation of related shares of expenses has low effect on value of optimal parameters *V, λ, W*, but considerably effects the total costs ΣC_i. It can be explained in such a way: the point of operation costs C_f is changed pretty much under changed of *V, λ, W*, than points C_e and C_a. That is why C_f defined the curvature and extremum of curve $\Sigma C_i = f(V, \lambda, W,)$.

Nevertheless, despite on degree of approximation of such method, the optimal parameters of airship turned out sufficiently reliable. The value of total expenses gives possibility to compare the airship effectiveness with other types of aircraft and transports.

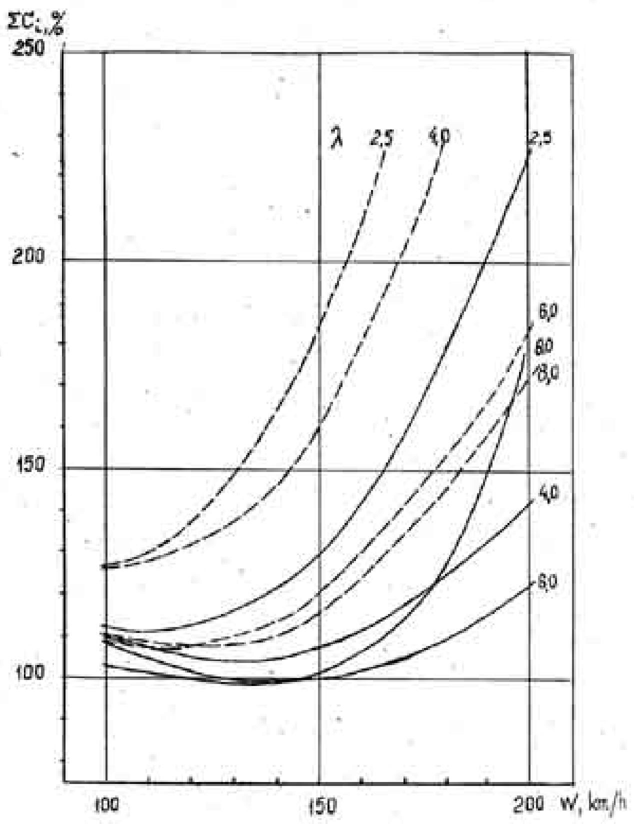


Fig.16. Related reduced annual expenses changing in dependence on W and λ under D_w max:

-------------------- oblong shape airship;

- - - - - - - - oblate shape airship.

For the given concrete example (Fig.16), optimal variant corresponding to minimum of reduced annual expenses is reached by extended shape airship with λ =6…8, volume V=140000 … 225000 m^3 under flying speed W=130 km/h.

Received optimal parameters for perspective vehicles with decreased weight under value C_x, decreased by 10% are similar to parameters of last generation Zeppelin's airship (LZ-129, LZ-130, "Akron"), (λ =6, V=200 000 m^3, W=130 km/h). We have every reason to presume that Zeppelin attained the optimal structures with maximum performances all in this power for those times.
Making perspective airships of large cargo capacity (payload) and heightened cargo turnover is inevitably bounded up with decreasing of fineness ratio λ and
increasing of total reduced annual expenses related per one ton-kilometer.
Universal algorithm of computer optimization of any airship type with different aim function can be made on the base of elaborated dependencies.

The comparison of airships having the same aim function magnitude (W=130 km/h, L=1200 km). Table 5.

NN	V, m^3	λ	G_P, t	$G_P \cdot n_{fl}$, t/d	$\overline{g}$, kg / t · km
1	225 000	6,0	94,396	147,257	0,07783
2	140 000	8,0	62,835	98,023	0,09270